Homework Set 7

(sect 4.1 – 4.3)

1. Let V be the set of points inside and on the unit circle in the xy-plane. That means, let $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + x^2 \leq 1 \right\}$. Find a specific example—two vectors or a vector and a scalar—to show that V is not a subspace of $\mathbb{R}^2$.

2. Let W be the set of all vectors of the form $\begin{bmatrix} 2t \\ 0 \\ -t \end{bmatrix}$. Show that W is a subspace of $\mathbb{R}^3$.

3. Let H be the set of all vectors of the form $\begin{bmatrix} 5b + 2c \\ b \\ c \end{bmatrix}$, where b and c are arbitrary real numbers. Find $\boldsymbol{u}$ and $\boldsymbol{v}$ such that $H = span\{\boldsymbol{u}, \boldsymbol{v}\}$. Why does this show that H is a subspace of $\mathbb{R}^3$?

4. Let $\boldsymbol{v_1} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\boldsymbol{v_2} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\boldsymbol{v_3} = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$, and $\boldsymbol{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$.

   a. Is $\boldsymbol{w}$ in $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$? How many vectors are in $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$?

   b. How many vectors are in $Span\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$?

   c. Is $\boldsymbol{w}$ in the subspace spanned by $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$? Why?

5. Let K be the set of all vectors of the form $\begin{bmatrix} 4a + 3b \\ 0 \\ a + b + c \\ c - 2a \end{bmatrix}$, where a, b, and c are arbitrary real numbers. Show that K is a subspace of $\mathbb{R}^4$, or give an example to show that K is not a subspace.

6. Determine if $w = \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}$ is in $Nul(A)$ (ie: the Null Space of A), where $A = \begin{bmatrix} 5 & 21 & 19 \\ 13 & 23 & 2 \\ 8 & 14 & 1 \end{bmatrix}$

7. Show that W is a vector space, or find a specific example which shows how W fails to be a vector space. $W = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{matrix} a + 3b = c \\ a + b + c = d \end{matrix} \right\}$

8. Find A such that the given set is $Col(A)$ (ie: the Column Space of A).
$\left\{ \begin{bmatrix} 2s + 3t \\ r + s - 2t \\ 4r + s \\ 3r - s - t \end{bmatrix} : r, s, and\ t\ are\ real \right\}$

9. Find k such that Nul(A) is a subspace of $\mathbb{R}^k$, and find l such that Col(A) is a subspace of $\mathbb{R}^l$, where $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 0 & -5 \\ 0 & -5 & 7 \\ -5 & 7 & -2 \end{bmatrix}$

10. Let $A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$. Determine if $\boldsymbol{w}$ is in $Col(A)$. Is $\boldsymbol{w}$ in $Nul(A)$?

For questions 11 and 12, state whether the sets are bases for $\mathbb{R}^3$ or not. If not, determine whether the set is linearly independent or if the set spans $\mathbb{R}^3$.

11. $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -7 \\ 5 \\ 4 \end{bmatrix}$

12. $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ -5 \\ 6 \end{bmatrix}$

13. Find a basis for the Null Space of the matrix: $\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 3 & -2 & 1 & -2 \end{bmatrix}$

14. Assume that A is row equivalent for B. Find the bases for $Nul(A)$ and $Col(A)$.

$A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}$, and $B = \begin{bmatrix} 1 & 0 & 6 & 5 \\ 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

15. Find a basis for the space spanned by the given vectors:

$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}$